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COMPARING OF NUMERICAL SCHEME TO THE METHODS EULER POLYGON HEUN'S WITH AN ANALYTICAL METHODS

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ABSTRACT

This paper describes the evaluation on numerical confusion over ternary regular finite equations about methods. Are rarely used, while the simple Euler's technique is popular amongst researchers. We compare four explicit methods: Euler's, Heun's methods, and polygon, including the honor analytical, is the accurate answer and value about external records regarding the simulation. The carelessness within analytical then numerical are moderate by way of the comparison along half previously worked. Euler technique has never been good including accordant models, polygon it turns outdoors in conformity with be the good scheme for much practical situations. The Heun's constantly prevails polygon method though both are over advance rule and has the identical scale of h= 0.25.

Key words: Euler Equation; Heun's Equation; Polygon Equation; Analytical Solutions; Initial Value.

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1. INTRODUCTION

With the advent regarding high-speed personal a reduce between the price of laptop assets within general (1), then numerical strategies pc modeling has ended up a vital section about the scientific method The third process according to studying physical problems, into culling regarding the theory among experimental, Methods numerical power according to remedy differential equations was lengthy into mathematicians then scientist lengthy before the emergence of modern computers (2). One regarding the oldest or simplest algorithms is Euler's method, also recognized as much the Euler-Kochi technique then the polygonal method. Where the virtue regarding x into the end targeted because of show h. Thus, the native calamity between Euler's techniques is into rule concerning h, as like a result into the

international error on the provision h. Therefore, Euler's algorithm is the forward order. It requires high exactness using a great algorithm in half instances and absolutely younger periods h (thus, also many steps) yet increase the volume tale day and rounding errors (3), The improved model of the Euler technique is obtained by storing ternary phrases In Taylor's enlargement characteristic as an alternative about two, who offers the second-ranking Algorithm (4). The accuracy concerning the Euler method can stay similarly accelerated consisting of Taylor's growth within numerical calculations. In its way, Different numerical strategies because of fixing a huge extent of issues with special orders on propriety and over a range of stages of complexity (5). Numerical solutions of normal differential equation ODEs have an Euler approach and Heun's, The Heun's differential equation (HDE) was introduced in 1889 by Karl M.W.L. Their methods are present to solve initial value problems, first-order Euler’s (6), first order Hen's and polygon method. The numerical results show the polygon method is more convergent then both methods. A new nonlinear adaptive numeric solution for an ordinary differential equation with initial conditions the main features is to implement nonlinear polynomial in a neural network-like adaptive frame work (7). There are comparative studies of numerical methods for the accuracy of the numerical solution is checked by comparing of the solution for the numerical methods functions and its derivatives into the obtained finite ordinary differential equations in the last section (8).

The purpose concerning the study is according to compare numerical methods that have much less linearity to a specific solution about the analytical solution, who is the method regarding numerical improvement that is, it achieves profitable results.

2. NUMERICAL METHODS

There are much specific numerical techniques in accordance with solve a broad measure about issues with one-of-a-kind resolution instructions at one of a kind tiers over complexity (9). For example, Numerical options because of regular deferential equation ODEs with the differential equation over unique equations, we perform have a simple plan over the Euler technique then Heun's diagram, polygon diagram because of evaluation together with the particular answer of the analytical method the use of perfect factor methods, we wish solely provide 3 numerical methods (10). These numerical methods are also through advice integration of the analytical along initial value (11).

2.1. Euler's method

Euler method is one of the oldest numerical methods used for integrating the ordinary differential equation. Thought this method is not used in practice, its understanding will help us to gain insight into the nature of predictor-corrector methods (12).

Consider the differential equation of first order

$$\frac{dy}{dx} = f(x,y) with\ initial\ condition f(to) = yo$$

2.2. Heun Method

Euler method is the simplest of all one-step methods. It does not require any differentiation and is easy to implement on computers. However, its major weakness is large truncation errors. This is due to its linear character-rustic. Recall that Euler method uses only the first two terms of the Taylor series. In Euler method, the slope at the beginning of the interval is used to extrapolate yi to yi+1 over the entire interval (13). Thus

$$yi + 1 = yi + m1h$$

Where m1is the slope at (xi, yi).

2.3. Polygon Method

Another modification of Euler method is to use the slope of the function at the estimated midpoints of (xi, yi) and (xi + 1, yi + 1) to approximate yi + 1. (14) Thus

$$= \ yi + f\left(xi + \frac{h}{2} + \frac{\Delta y}{2}\right), yi + \frac{dy}{2})h$$

Δy Is the estimated incremental value of y from yi and can be obtained using Euler's formula as

$\Delta y = hf\ (xi, yi)$

Then, equation $yi + 1 = yi + f\left(xi + \frac{h}{2}, yi + \frac{dy}{2}\right)h$ can be written as

$$yi + 1 = yi + hf\left(xi + \frac{h}{2}, yi + \frac{h}{2f}\right)(xi, yi)$$

$$= (yi + f\left(xi + \frac{h}{2}, yi + \frac{m1}{2}h\right)$$

$$yi + \frac{m2}{\mathrm{h}}$$

Where

$m1 = f\ (xi, yi)$ And $m2 = f\ \left(xi + \frac{h}{2}, yi + \frac{m1}{2}\right)$

An Equation above is known as the modified Euler method or improved polygon method.

3. NUMERICAL RESULT

Euler method: Example 1. Solve the Given equation by Euler

$$y(x) = \frac{2y}{x}$$

Y =f(x, y) = 2x/y. For x_o =1, y = 2, h = 0.25

Using Euler method

Y (1) = 2

Y (1.25) = yi+ y0 + hf (x, y)

Y (1.25) = 2 + 0.25 $\frac{2x2}{1}$ = 3.00

Y (1.50) = 3.0 + 0.25 $\frac{2x\ 3.00}{1.25}$ = 4.2

Y (1.75) = 4.2 + 0.25 $\frac{2x4.2}{1.50}$ = 5.6

Y (2.0) = 5.6 + 0.25 $\frac{2x5.6}{1.75}$ = 7.2

Heun method: Example 2. Solve the equations using heun.

Iteration 1

M_1 = $\frac{2x2}{1}$ = 4.0

Y (1.25) = 2+ 0.25(4.0) = 3.0

M_2 = $\frac{2x3.0}{1.25}$ = 4.8

Iteration 2

M_1 = $\frac{2x2.0}{1.25}$ = 4.96

Y (1.5) = 3.1 + 0.25(4.96) = 4.34

$M_2 = \frac{2x4.24}{1.5}$ = 4.8

Y (1.5) = 3.1 + $\frac{0.25}{2}$ (4.96 + 5.79) = 4.44

Iteration 3

$M_1 = \frac{2x4.44}{1.5}$ = 5.92

Y (1.75) = 4.44 + 0.25(5.92) = 5.92

$M_2 = \frac{2x5.92}{1.75}$ = 6.77

Y (1.75) = 4.44 $\frac{0.25}{2}$ (5.92 + 6.77) = 6.03

Iteration 4

$M_1 = \frac{2x6.03}{1.75}$ = 6.89

Y (2.0) = 6.03 + 0.25(6.89) = 7.75

$M_2 = \frac{2x7.75}{2}$ = 7.75

Table.1 Euler and Heun's result.

X	Eular's method	Heun's method
1.00	2.00	2.00
1.25	3.00	3.10
1.50	4.20	4.44
1.75	5.60	6.03
2.00	7.20	7.86

At first step, Euler and Heun's have same value but from second till fifth step Heun's distinction over Euler it has less iteration it clear that Heun's the best at this table.

Polygon method: Example 3. Solve the equation using

$y\,(1) = \ 2$

$y\,(1.25) = \ 2 + 0.25\ f\ (1 + 0.125, 2 + 0.125\ f(1, 2)$

$= \ 2 + 0.25f\ (1.125, 2.5) = 3.11$

$y\,(1.5) = 3.11 + 0.25f\ (1.25 + 0.125, 3.11 + 0.125\ f\ (1.25, 3.11)$

$= \ 3.11 + 0.25f\ (1.375,3.731) = 4.47$

$y\,(1.75) = 4.47 + 0.25f\ (1.5 + 0.125, 4.47 + 0.125\ f\ (1.5, 4.47)$

$= \ 4.47 + 0.25f\ (1.625, 5.215) = 6.074$

$y\,(2.00) = 6.07 + 0.25f\ (1.75 + 0.125, 6.07 + 0.125\ f\ (1.75, 6.07)$

$= \ 6.07 + 0.25f\ (1.875, 7.027) = 7.94$

Analytical solution: Example 4.Solve the equation analytical

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\frac{dy}{y} = \frac{2dx}{x}$$

$$\ln y = 2\ln x + c$$

$$\ln y = \ln x^2 + c$$

$\ln y - \ln x^2 = c$

$\ln \frac{y}{x^2} = c$

$\frac{y}{x^2} = e^c = c1$

$y = 2, \; x = 1$

$\frac{2}{1^2} = c1$

$c1 = 2$

$y = 2$

$y = 2x^2$

$when\ x1 = 1, \qquad h = 0.25$

$y = 2$

$when\ x2 = x1 + h, \qquad h = 0.25$

$x2 = 1 + 0.25 = 1.25$

$y = 2(1.25)2$

$y = 3.125$

$when\ x3 = x2 + h, \qquad h = 0.25$

$x2 = 1.25 + 0.25$

$x2 = 1.50$

$y = 4.50$

$when\ x4 = x3 + h, \qquad h = 0.25$

$x4 = 1.50 + 0.25$

$x4 = 1.75$

$y = 2(1.75)^2$

$y = 6.125$

$when\ x5 = x4 + h, \qquad h = 0.25$

$x5 = 1.75 + 0.25$

$x5 = 2$

$y = 2(2)^2$

$\mathrm{y} = 8$

Table. 2 Heun's and polygon result

Heun's method	Polygon method	Analytical
2.00	2.00	2.00
3.10	3.11	3.125
4.44	4.47	4.50
6.03	6.07	6.125
7.86	7.94	8.00

Now we compare between Heun's and polygon methods, clear that at first step both methods have the same value but from the second polygon step to fifth polygon step have more accuracy than Hen's, it has less iteration from Heun's at this table evidently polygon is the win method.

Table. 3 The result to exact value of y(x) and the estimated values by both the method are tabulated below.

X	Eular's method	Heun's method	Polygon method	Analytical
1.00	2.00	2.00	2.00	2.00
1.25	3.00	3.10	3.11	3.125
1.50	4.20	4.44	4.47	4.50
1.75	5.60	6.03	6.07	6.125
2.00	7.20	7.86	7.94	8.00

The result of Euler, Heuns, polygon, analytical method, it clear polygon is optimum the best convergent from all methods, have good approximate and less iteration to analytical result, also has smaller error to analytical from the other numerical methods, get smaller error with polygon we find the numerical approximate win with a polygon (15).

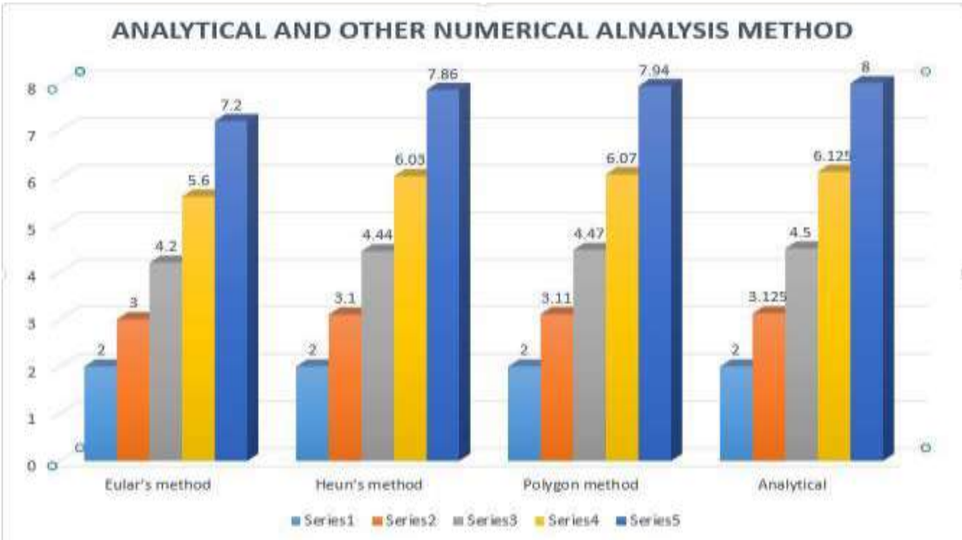

Figure 1 Comparing of numerical methods with the analytical solution [17].

Comparison of results obtained using Euler methods with Hen's. Heuns method is also summarized for several different cell sizes h = 0.25 Table 1. When x = 1 the results are all. The methods listed in the table are exactly for each other, a decimal single number. When the cell size is determined as 0: 1, the results obtained from Heuns are equal to one, Also the Heun's results are better than Euler, have more accurate to analytical solutions at all stages. The comparison of the Heuns method with the polygon meets with the polygon only in the first step, and also with the remaining phases having a decimal error (16), less than one. Comparison of the last polygon with the real solution of the analytical method at the confluence at the first step, and in step five differ from six decimal numbers (17). We can see the accuracy level of all methods shown in Figure 1.

4. CONCLUSIONS

This article was compared with the problem of numerical methods in the ordinary differential equation (ODE) on the behavior of error of numerical methods, which can be modeled by numerical comparison. The calculation method of numerical Eller with the help of Heun's control solution with initial value setting gives acceptable accuracy. This reality has been demonstrated by three methods: Euler, Hen's, Polygon, and Performance Accuracy in this article. We note that the approximate analytical solutions for four methods at the confluence in the first step only with numerical scalar solutions using mathematics and Mathcad are comparable. It can be performed using programs such as Mathematica, or Matlab. As a result

of the calculations, the researchers found that when calculating the numerical methods with the Heun's in the first step, the accuracy of the task is h = 0.25. This is due to very large values of derivatives that occur near this point. The Heun's method of converting variables has been developed, which calculates numerical solutions by polygon method. Therefore, the numerical values of polygons obtained in confidence in many stages can be supplemented using comparison with the analytical solution. Get the minimum error from Heun's method of existing methods, so that both roads converge. Numerical results are accurate because the numerical solution is close to the exact solution. Heun's algorithm error is less than Euler. Both Heun's and polygon are advantageous, and a polygon is the winner of analytical results.

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